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Frequency-sensitive stochastic resonance in periodically forced and globally coupled systems

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Abstract. A model of globally coupled bistable systems consisting of two kinds of sites, subject to periodic driving and spatially uncorrelated stochastic force, is investigated. The extended system models the competing process of activators and suppressers. Analytical computations for linear response of the system to the external periodic forcing is carried out. Noise-induced Hopf bifurcation is revealed, and stochastic resonance, sensitively depending on the frequency of the external forcing, is predicted under the Hopf bifurcation condition. Numerical simulations agree with the analytical predictions satisfactorily.

PACS. 05.45.+j Fluctuation phenomena, random processes, and Brownian motion

1 Introduction

Usually one thinks that noise may play role to spoil signal, reduce coherence and increase disorder. However, it is found that under proper nonlinear condition noise can play rather positive role in improving output signal, enhancing coherence. In other words, increasing noise (increasing disorder) in the input may result in increasing order in the output. This seemly striking feature of nonlinear stochastic systems is termed as stochastic resonance (SR). The topic of SR has attracted much attention for nearly two decades [1–8], for the latest review, see reference [9]. The investigation has focused on the "resonance" with respect to noise intensity. Recently, there has been a great interest to extend the SR study to coupled systems [10–20]. In reference [19], the authors suggested a model of globally coupled overdamped nonlinear oscillators with two kinds of competing cells. The most interesting feature of this model is that, on one hand, a resonance purely noise induced (*i.e.*, can be called stochastic resonance) can be found, on the other hand, this resonance has sensitive frequency dependence (*i.e.*, the meaning of conventional resonance in physics is recovered). This SR feature is identified at a noise-induced Hopf bifurcation point. Thus, the scope of the investigation of SR can be considerably enlarged. This paper is essentially developed from reference [19]. First, analytical predictions are carried out, based on the linear response theory. Second, numerical simulations for the original coupled systems fully confirm analytical predictions. The model investigated in this paper reads

$$\dot{x_i}= a_1x_i - b_1x_i^3 + \mu_1 Z(t) + A_1\cos(\Omega t+\gamma_1) + \Gamma_i(t)$$

$$\dot{y_i}= a_2x_i - b_2y_i^3 + \mu_2 Z(t) + A_2\cos(\Omega t+\gamma_2) + \Delta_i(t)$$

$$\langle \Gamma_i(t)\rangle = 0, \quad \langle \Gamma_i(t)\,\Gamma_j(t')\rangle = 2D_1\delta_{ij}\delta(t-t')$$

$$\langle \Delta_i(t)\rangle = 0, \quad \langle \Delta_i(t)\,\Delta_i(t')\rangle = 2D_2\delta_{ij}\delta(t-t'),$$

$$\langle \Gamma_i(t)\Delta_j(t)\rangle = 0 \tag{1.1}$$

where all parameters $a_{1,2},\ b_{1,2}$ and $\mu_{1,2}$ are positive, and

$$Z(t) = X(t) - Y(t), \quad X(t) = \frac{1}{N}\sum_{k=1}^{k=N} x_k,$$

$$Y(t) = \frac{1}{N}\sum_{k=1}^{k=N} y_k. \tag{1.2}$$

The system consists of two kinds of space cells x_i, $i = 1, 2, \cdots, N$, and $y_j,\ j = 1, 2, \cdots, N$. The inner dynamics of each cell is governed by a bistable system forced by a periodic injection and a white noise driving. All cells are coupled to each other through the mean field $Z(t) = \frac{1}{N}\left(\sum_{i=1}^{i=N} x_i - \sum_{j=1}^{j=N} y_j\right)$ where x cells are regarded

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active while y cells are suppressive. The idea of competition between activators and suppressers appears in many fields, and this idea is crucial in our case for the new feature of SR with sensitive frequency dependence.

2 Analytical prediction of SR with sensitive frequency dependence

For treating equations (1.1) we can start with analyzing the dynamics of single cell

$$\dot{x} = ax - bx^3 + A\cos(\Omega t) + \Gamma(t) \qquad (2.1)$$

which can be transformed to a Fokker-Planck equation (FPE)

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[ax - bx^3 + A\cos(\Omega t)\right] P(x,t) + D\frac{\partial^2}{\partial x^2}P(x,t). \qquad (2.2)$$

At small forcing $A \ll 1$, the asymptotic solution $\langle x(t)\rangle$ of is known to be [6]

$$\begin{aligned}\langle x(t)\rangle &= \mathrm{Re}\left\{B\exp\left[i(\Omega t+\phi)\right]\right\} \\ &= \mathrm{Re}\left\{AM\exp[i(\Omega t+\theta)]\right\}\end{aligned} \qquad (2.3)$$

$$M\exp(i\theta) = \sum_{n=1}^{n=\infty} g_n \exp(i\alpha)$$

$$g_n = \frac{1}{(\lambda_n^2+\Omega^2)^{\frac{1}{2}}}\langle n \mid x \mid 0\rangle \left\langle n \left| \frac{\partial}{\partial x} \right| 0 \right\rangle \qquad (2.4)$$

$$\cos(\alpha_n) = \lambda_n/\left(\lambda_n^2+\Omega^2\right)^{\frac{1}{2}},\ \sin(\alpha_n) = -\Omega/\left(\lambda_n^2+\Omega^2\right)^{\frac{1}{2}}$$

where $|n\rangle$ and $\langle n|$ are the nth right and left eigenvectors of the FP operator (2.2) with $A=0$, respectively, and the corresponding eigenvalues reads $-\lambda_n$, which are ordered as $0=\lambda_0<\lambda_1<\lambda_2<\cdots$

In equations (1.1) all the cells are coupled to each other *via* the same quantity $Z(t)$. In the asymptotic state, the motions of all x_i and y_i are characteristically identical for different i. Therefore, the $2N$ coupled Langevin equations can be transformed to two coupled Fokker-Planck equations

$$\begin{aligned}\frac{\partial P(x,t,Z(t))}{\partial t} &= -\frac{\partial}{\partial x}\left[a_1x - b_1x^3 + \mu_1 Z(t) + A_1\cos(\Omega t+\gamma_1) + D_1\frac{\partial}{\partial x}\right] P(x,t,Z(t)) \\ \frac{\partial P(y,t,Z(t))}{\partial t} &= -\frac{\partial}{\partial y}\left[a_2y - b_2y^3 + \mu_2 Z(t) + A_2\cos(\Omega t+\gamma_2) + D_2\frac{\partial}{\partial y}\right] P(y,t,Z(t))\end{aligned} \qquad (2.5)$$

where x and y represent the variables x_i and y_i, respectively. Now the remaining point of (2.5) is to close the equations by determining the coupling quantity $Z(t)$ (*i.e.*, $X(t)$ and $Y(t)$). For doing this we use self-consistent conditions

$$\langle x(t)\rangle = X(t), \qquad \langle y(t)\rangle = Y(t) \qquad (2.6)$$

where $\langle x(t)\rangle$ and $\langle y(t)\rangle$ are the statistical averages of x and y from (2.5) $\langle x(t)\rangle = \int xP(x,t)dx$, $\langle y(t)\rangle = \int yP(y,t)dy$, while $x(t)$ and $y(t)$ are the space averages of x_i and y_i as $X(t) = \frac{1}{N}\sum_{i=1}^{i=N} x_i(t)$ and $Y(t) = \frac{1}{N}\sum_{i=1}^{i=N} y_i(t)$. The identities of (2.6) are valid in the large system size limit due to the symmetry between different cells in the asymptotic state and the Large Number Theory. It is emphasized as $N\to\infty$ the quantities $X(t)$ and $Y(t)$ (also $Z(t)$) are no longer stochastic, their fluctuations are canceled by the spatial averages over the infinite number of cells.

With equations (2.5, 2.6) we can compute $X(t)$ and $Y(t)$, based on the solution (2.3, 2.4). With small forcing ($A \ll 1$) we can separate both $X(t)$ and $Y(t)$ to two parts

$$X(t) = X_0 + X_t, \quad Y(t) = Y_0 + Y_t$$

where X_0 and Y_0 are the solutions of equations (2.5) at $A_{1,2}=0$, which are set to zero, $X_0 = Y_0 = 0$, in our discussion. X_t and Y_t are assumed to be

$$X_t = B_1\cos(\Omega t+\phi_1), \quad Y_t = B_2\cos(\Omega t+\phi_2)$$

with B_1, B_2 and ϕ_1, ϕ_2 to be determined. Inserting X_t and Y_t into equations (2.5), and considering the solution of (2.3) we can self-consistently arrive at two coupled algebraic equations

$$\begin{aligned}B_1\exp(i\phi_1) &= \mu_1 M_1 \exp(i\theta_1)\left[B_1\exp(i\phi_1) - B_2\exp(i\phi_2)\right] + A_1M_1\exp\left[i(\theta_1+\gamma_1)\right] \\ B_2\exp(i\phi_2) &= \mu_1 M_2 \exp(i\theta_2)\left[B_1\exp(i\phi_1) - B_2\exp(i\phi_2)\right] + A_2M_2\exp\left[i(\theta_2+\gamma_2)\right]\end{aligned} \qquad (2.7)$$

where the quantities $M_{1,2}$ and $\theta_{1,2}$ are given in equations (2.4) by setting $Z=0$ in equations (2.5). Equations (2.7) have solutions

$$B_{1,2}\exp(i\phi_{1,2}) = \frac{Q_{1,2}}{Q_0},$$

$$Q_0 = 1 - \mu_1 M_1\exp(i\theta_1) + \mu_2 M_2 \exp(i\theta_2)$$

$$\begin{aligned}Q_{1,2} = {} & A_{1,2}M_{1,2}\exp\left[i(\gamma_{1,2}+\theta_{1,2})\right] \\ & \pm M_1M_2\exp\left[i(\theta_1+\theta_2)\right]\left[A_{1,2}\mu_{2,1}\exp(i\gamma_{1,2}) - A_{2,1}\mu_{1,2}\exp(i\gamma_{2,1})\right].\end{aligned} \qquad (2.8)$$

An obvious conclusion from (2.8) is that the coherent output ($B_{1,2}$) can be greatly enhanced at the vanishing Q_0. The resonance condition is determined by $Q_0 = 0$ as

$$\begin{aligned}\mu_1M_1\cos(\theta_1) &= \mu_2M_2\cos(\theta_2) + 1, \\ \mu_1M_1\sin(\theta_1) &= \mu_2M_2\sin(\theta_2).\end{aligned} \qquad (2.9)$$

It is striking that the stochastic resonance condition and the linear response solution of $2N$ coupled Langevin equations can be exactly computed by two enormously simplified coupled algebraic equations in the large system size limit, the coefficients of the two algebraic equations can be given by analyzing a one-dimensional FPE. It is emphasized that the solutions (2.8) are valid for arbitrary D_i, μ_i, and Ω. The only restriction is $A \ll D_i$ and 1. For $A \ll D_{1,2} \ll 1$, we can keep only the first term in (2.4), $M = g_1$, and $\theta = \alpha_1$, and obtain

$$M_{1,2} = \frac{a_{1,2}\lambda(1,2)}{b_{1,2}D_{1,2}\sqrt{\lambda(1,2)^2 + \Omega^2}}$$

$$\cos(\theta_{1,2}) = \frac{\lambda(1,2)}{\sqrt{\lambda(1,2)^2 + \Omega^2}},$$

$$\sin(\theta_{1,2}) = \frac{-\Omega}{\sqrt{\lambda(1,2)^2 + \Omega^2}} \qquad (2.10)$$

where $\lambda(1,2)$ are the first nonzero eigenvalues computed from the first and second FPEs of (2.5), respectively.

$$\lambda(1,2) = \frac{\sqrt{2}a_{1,2}}{\pi b_{1,2}} \exp\left(-\frac{a_{1,2}^2}{4b_{1,2}D_{1,2}}\right). \qquad (2.11)$$

Afterwards we simply denote $\lambda(1,2)$by $\lambda_{1,2}$, respectively. Inserting (2.10) to (2.9) we can explicitly present the SR conditions as

$$\lambda_1 + \lambda_2 = \frac{a_1\mu_1\lambda_1}{b_1D_1} - \frac{a_2\mu_2\lambda_2}{b_2D_2} \qquad (2.12)$$

$$\Omega^2 = \frac{(\lambda_1 - \lambda_2)\left(\frac{a_1\mu_1}{b_1}\lambda_1 + \frac{a_2\mu_2}{b_2}\lambda_2\right) - \left(D_1\lambda_1^2 + D_2\lambda_2^2\right)}{D_1 + D_2}. \qquad (2.13)$$

In the case of

$$\frac{a_1\mu_1}{b_1} = \frac{a_2\mu_2}{b_2} = \mu, \quad D_1 = D_2 = D \qquad (2.14)$$

we get rather compact SR conditions as

$$\mu_h = \frac{D(\lambda_1 + \lambda_2)}{\lambda_1 - \lambda_2} \qquad (2.15a)$$

$$\Omega^2 = \lambda_1\lambda_2. \qquad (2.15b)$$

A physically meaningful conclusion is that SR occurs at the frequency of geometrical mean of the decay rates of the two bistable systems. In Figures 1 we plot the SR conditions of equations (2.15) in $D - \mu$ plane and $D - \Omega$ plane at $a_2 = b_{1,2} = 1$ and $a_1 = 0.9$ (these parameters are used afterwards).

A striking feature of (2.8) is that there exists a divergence in the linear response at the SR condition; this is essentially distinguished from the SR for single bistable systems where no divergence can be associated with the stochastic resonance. Similar divergence feature is also found in reference [10]. Divergence of linear response is believed to be related to phase transition phenomena [10,13]. Phase transitions are found indeed in reference [10] and in our cases. In the former it is spontaneous ordering phase transition, pitchfork bifurcation, while in the latter case it is spontaneous oscillation phase transition, Hopf bifurcation. We would like to emphasize that in both cases the phase transitions are purely noise-induced. Therefore, a sharp peak of output signal (resonance) can appear at an optimal noise intensity when we vary the input noise; that is what one refers commonly to stochastic resonance. If we plot the amplitude of the output signal *vs.* the frequency of the input signal, in reference [10] one find the peak at $\Omega = 0$; this feature is observed so far in all previous works on SR problem. In our case the peak can appear at nonzero Ω, namely, a SR with sensitive frequency dependence is justified. The reason for the difference between the system of reference [10] and the present system is that in the former case the spontaneous ordering bifurcation is associated to frequency $\Omega = 0$ while at the noise-induced Hopf bifurcation in the latter case a nonzero inner frequency appears, which can be resonant with an external force with the equal frequency. It is interesting to notice that the SR with frequency sensitivity in our system

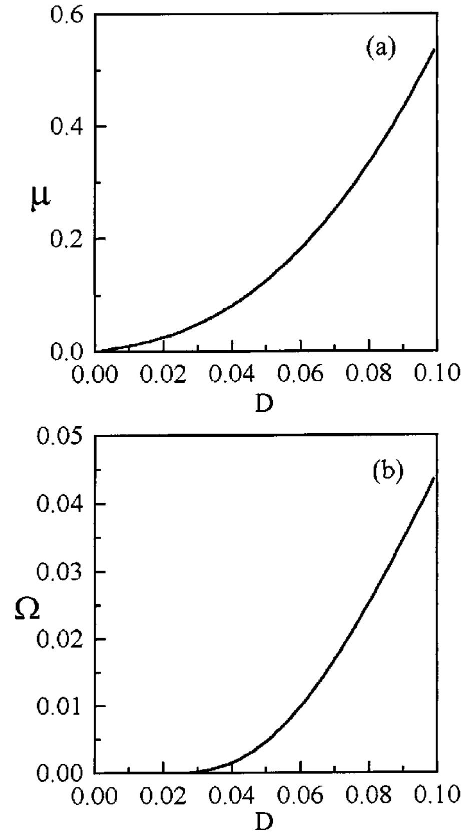

Fig. 1. $a_1 = 0.9$, $a_2 = b_1 = b_2 = 1$, $A_1 = A_2 = A$, $D_1 = D_2 = D$, $a_1\mu_1 = \mu_2 = \mu$ (these parameter values and relations are used in all the following figures). $A = 0$. (a) μ_h *vs.* D according to equation (2.15a). (b) Ω *vs.* D according to equation (2.15b).

is due to the competition between the activators and suppressers, which is represented by the negative signs in the r.h.s. of equations (2.7) before the terms $B_2 \exp(i\phi_2)$. If we cancel the competition by changing these "-" signs to "+" (*i.e.*, use $Z(t) = X(t)+Y(t)$ instead of $Z(t) = X(t)-Y(t)$ in Eqs. (1.1)), the noise-induced Hopf bifurcation and the associated SR with sensitive frequency dependence can definitely disappear.

In the next section we will perform numerical simulations on the original equations (1.1), and verify the theoretically predicted SR conditions (2.15), and reveal the existence of SR with sensitive frequency dependence.

3 Numerical results

For understanding the basic dynamics of the unforced system, we run equations (1.1) by setting $A_1 = A_2 = 0$, and present the results in Figures 2. Figures 2a, b and c plot the data for $N = 400$ after transient process. For $\mu < \mu_h$ ($\mu_h \approx 0.2$ for $D = 0.06$, see Fig. 1a) , we find a small spot around the origin in (a), which represents the stable stationary solution at $X(t) = Y(t) = 0$ (that verifies the assumption $X_0 = Y_0 = 0$ in the previous section). The finite spot area is due to the fluctuation caused by finite system size, the width of this area can be roughly estimated by applying the Large Number Theory $\Delta S_w \propto \frac{2\sqrt{3}}{\sqrt{N}} \approx 0.18$. For $\mu = \mu_h$, we find a typical feature of critical fluctuation amplification of Hopf bifurcation. At the Hopf bifurcation, the fluctuation can be also roughly estimated as $S_c \propto \frac{2\sqrt{3}}{\sqrt[4]{N}} \approx 0.8$. Both S_w and S_c, theoretically estimated, qualitatively agree with the numerical observations in Figure 2a and b. In Figure 2c we take $\mu > \mu_h$, a limit cycle solution is observed. All these observations of equations (1.1) are fully consistent with the phase diagram of Figure 1a.

In Figures 3, 4 and 5, we plot the quantity $\beta = \frac{B_1}{A_1}$ against various control parameters. β represents the signal amplification, and B_1 is numerically computed as

$$B_1 = \frac{1}{T}\sqrt{\left[\int_0^T X(t)\cos(\Omega t)dt\right]^2 + \left[\int_0^T X(t)\sin(\Omega t)dt\right]^2} \tag{3.1}$$

where we take $T \gg \frac{2\pi}{\Omega}$ ($T = 50$ in our simulations). Each black disk represents a date obtained by averaging 50 runs of equations (1.1).

In Figure 3 we take $\mu = 0.2$, $\Omega = 0.012$, and plot β *versus* D for various A ($A_1 = A_2 = A$ is taken in all Figs. 3, 4 and 5). A stochastic resonance in conventional sense is identified. There exists an optimal noise intensity, at which the output takes maximum. The SR parameters $\mu = 0.2$, $\Omega = 0.012$ and $D \approx 0.06$ are identical to what are predicted in Figures 1a and b.

In Figure 4, we fix $D = 0.06$, $\Omega = 0.012$ and plot β *vs.* μ for different A, we also find peaked response curves. These peaks are well known for periodically forced coupled Brownian motions (see Ref. [10]).

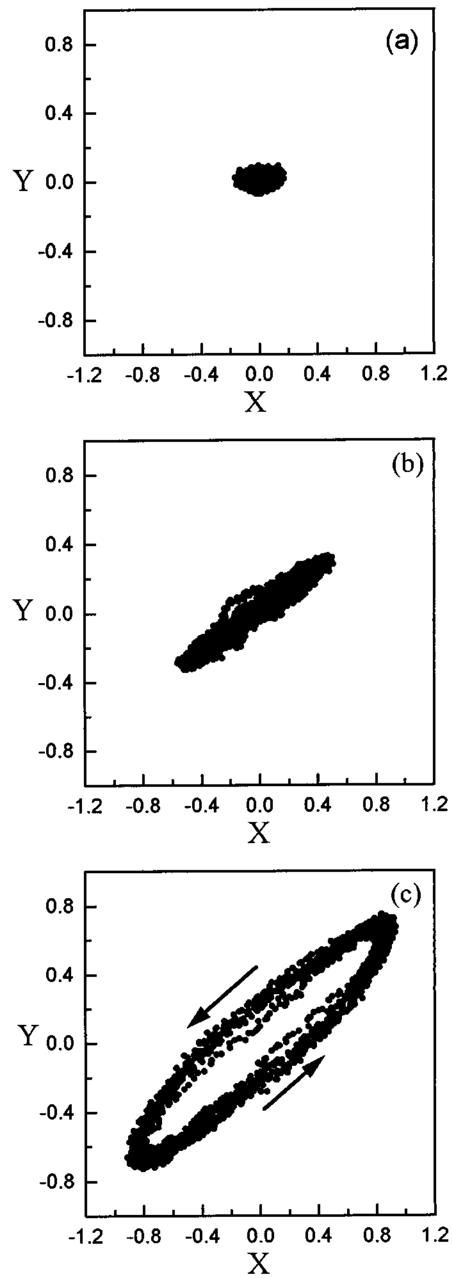

Fig. 2. $A = 0$, $D = 0.06$, $N = 400$. (a), (b) and (c): Asymptotic states obtained by numerically running equations (1.1) for different μ. (a) $\mu = 0.05 < \mu_h$. (b) $\mu = 0.2 \approx \mu_h$. A critical fluctuation amplification at the Hopf bifurcation point is observed. (c) $\mu = 0.45 > \mu_h$. A limit cycle solution appears after Hopf bifurcation.

The most interesting numerical results in this paper are presented in Figures 5 where β is plotted *vs.* Ω for $\mu = 0.2$, $D = 0.06$ and various A. We find nice SR curves peaked at finite frequency; that is different from the conventional SR behavior (where the peak is always located at $\Omega = 0$), but is in agreement with the conventional

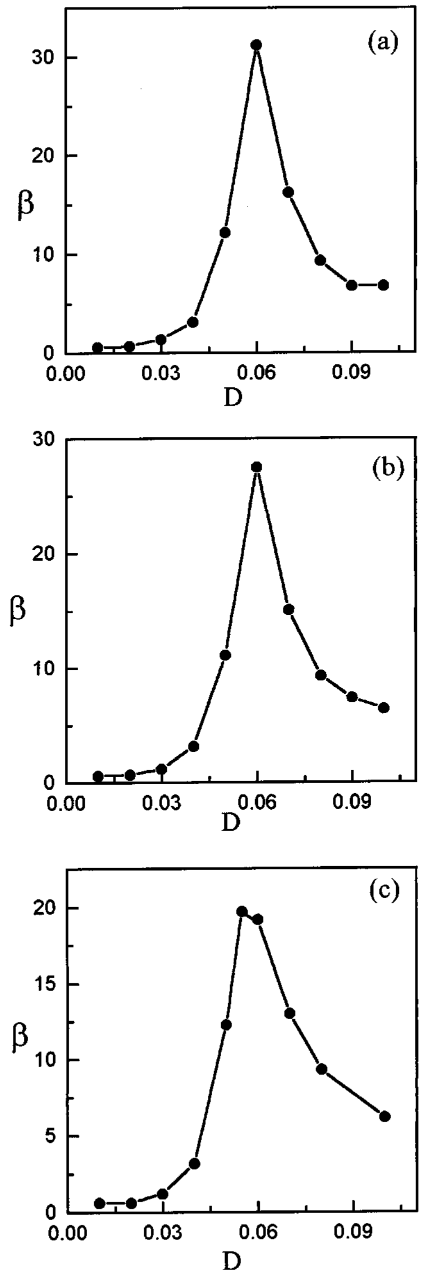

Fig. 3. $N = 400$, $\mu = 0.2$, $\Omega = 0.012$. β *vs.* D for different A. SR in conventional sense is observed. Disked curves are plotted by averaging data of 50 runs of equations (1.1) (the same meanings are kept in Figs. 4 and 5). (a) $A = 0.01$. (b) $A = 0.02$. (c) $A = 0.04$.

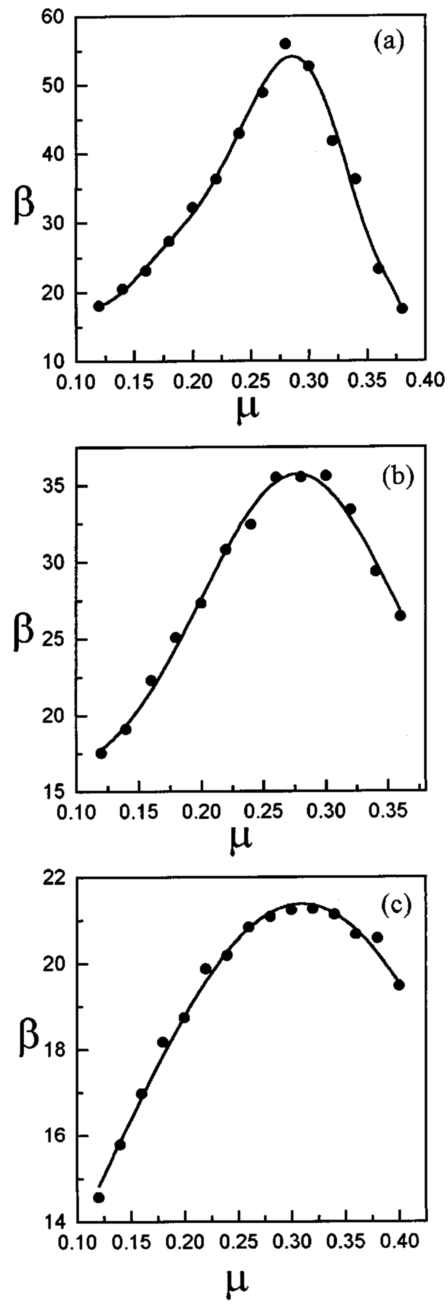

Fig. 4. $N = 400$, $D = 0.06$, $\Omega = 0.012$. β *vs.* μ for different A. (a) $A = 0.01$. (b) $A = 0.02$. (c) $A = 0.04$.

resonance behavior in physics. The reason for this sensitive frequency dependence of the SR responses can be easily understood from the unperturbed system ($A_1 = A_2 = 0$ in Eqs. (1.1)), where an inner oscillation with finite frequency appears *via* Hopf bifurcation; and at the bifurcation point a periodic input with the resonant frequency can be most effectively amplified. Two important points should be emphasized for this effect. First, this oscillation is purely noise induced (without noise, the coupled bistable systems of (1.1) have no any oscillations for such small μ), then the resonance is truly a stochastic resonance. Second, the resonance appears between the inner frequency and the external frequency, then this is truly a resonance in physically conventional sense; that essentially differs from the SR we know so far.

In both Figures 4 and 5 the parameters for resonance agree qualitatively with those theoretically predicted in Figure 1. Nevertheless, some quantitative deviations can be observed. The reason for these dismatches can be

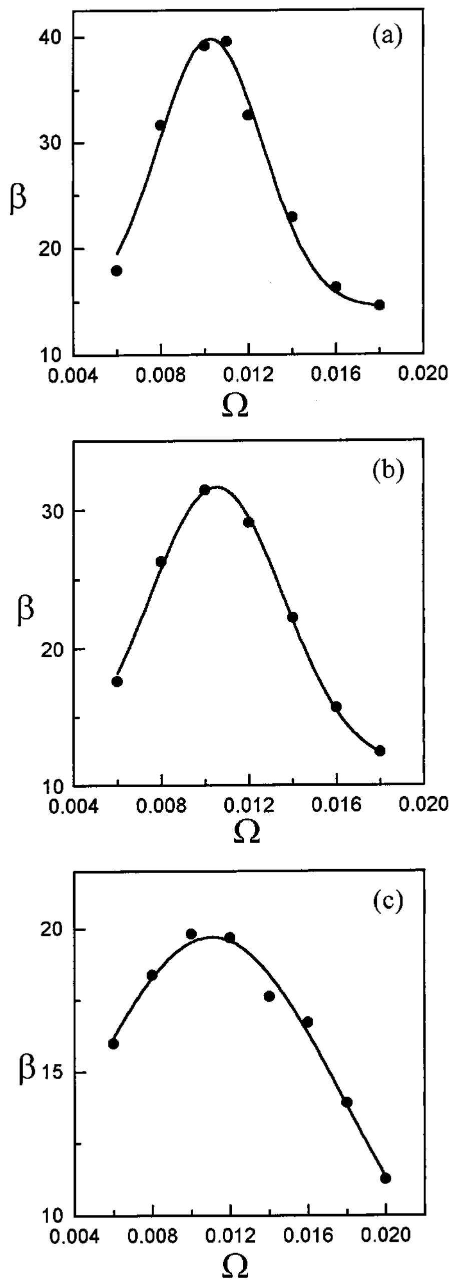

Fig. 5. $N = 400$, $\mu = 0.2$, $D = 0.06$. β *vs.* Ω for different A. SR with sensitive frequency dependence, a resonance in physically conventional sense (but not in conventional SR sense), is observed. (a) $A = 0.01$. (b) $A = 0.02$. (c)$A = 0.04$.

explained by finiteness of the quantities N, D, μ and A since the formulas (2.15) turn to be exact only in the limits $N \to \infty$, $A \to 0$, $D \to 0$, $\mu \to 0$ and $A \ll D$.

4 Conclusion

In conclusion we would like to make a few remarks. We have suggested a model of globally coupled overdamped oscillators, each site subjects to a spatially coherent and time periodic forcing and a spatially uncorrelated noise driving. All the sites are divided to two groups, one activators, and the other suppressers. The competition between these two kinds of sites is the key point for the essentially new results in this paper.

Without the periodic forcing, this model allows noise-induced oscillation with a finite characteristic frequency through Hopf bifurcation. Then with the periodic forcing one can find, at the Hopf bifurcation point, a resonance between the frequency of external force and that of the inner noise-induced oscillation; this leads to a SR with sensitive frequency dependence.

A well known self-consistent field approach is applied for analytically treating the high-dimensional globally coupled systems. According to the linear response theory, the SR condition of this $2N$-dimensional Langevin equations can be determined exactly, in the large system size limit, by two coupled algebraic equations with coefficients given from the analysis of a simple one-dimensional Brownian motion. The analytical predictions from these enormously simplified equations are satisfactorily confirmed by numerical simulations.

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